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Summary

Dancing with the Stars (DWTS) combines judges' scores with audience votes, but the vote counts are not disclosed. This creates an inverse problem: infer weekly fan vote shares consistent with observed eliminations. We develop a fan-vote estimation pipeline that (i) reconstructs a contestant–week panel, (ii) builds an interpretable prior intensity from celebrity/pro-dancer fan "stocks" and performance-driven vote surges, and (iii) corrects weekly vote shares to satisfy season-era elimination rules (rank-based in Seasons 1–2, percentage-based in Seasons 3–27, and bottom-2 lock in Seasons 28–34). For uncertainty, we construct a Dirichlet posterior centered at the point estimate, with a hierarchical prior on concentration to stabilize week-to-week variability, and report posterior consistency probabilities. A genetic algorithm further tunes hyperparameters by maximizing Bayesian consistency while penalizing uncertainty and failures.

Using the estimated fan votes, we compare the ranking and percentage scoring methods across all elimination weeks by cross-applying both rules. The two methods disagree in 28.09% of elimination weeks, indicating that rule choice materially changes outcomes. The percentage method aligns substantially better with historical eliminations than the rank method. We also quantify judge–fan disagreement (controversy) and analyze representative cases that highlight how fan support can offset technical deficits under different aggregation rules. Finally, we evaluate professional-dancer and celebrity-feature effects on judges and fans, showing that professional identity remains a first-order driver beyond celebrity attributes.

Key Words:

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1 Professional Dancer and Celebrity-Feature Effects on Judges and Fans

1.1 Data and outcomes

Let i index pairs (celebrity–professional), (s, w) index season and week, and let $\mathcal{A}_{s,w}$ be the set of active pairs in week w of season s. We analyze the merged panel of size $n = 2402$ complete cases.

Judges. Let $J_{i,s,w} \in (0, 1)$ be the within-week judge share (`judge_percent`). Define

$$\tilde{J}_{i,s,w} = \text{logit}(J_{i,s,w}), \qquad y^{(J)}_{i,s,w} = \tilde{J}_{i,s,w} - \frac{1}{|\mathcal{A}_{s,w}|} \sum_{k \in \mathcal{A}_{s,w}} \tilde{J}_{k,s,w}. \tag{1}$$

Thus $y^{(J)}$ measures *relative* judge advantage within each (s, w), removing week-level shocks.

Fans (components). From `dwts_fan_vote_estimates_components.csv`, we use the estimated vote-share components: $P^{(\text{tot})}$, $P^{(\star)}$ (celebrity fan-base), $P^{(\text{pro})}$ (pro-fixed), and $P^{(\text{perf})}$ (performance-induced). For $o \in \{\text{tot}, \star, \text{pro}\}$ we define

$$\tilde{P}^{(o)}_{i,s,w} = \text{logit}\Big(P^{(o)}_{i,s,w}\Big), \qquad y^{(o)}_{i,s,w} = \tilde{P}^{(o)}_{i,s,w} - \frac{1}{|\mathcal{A}_{s,w}|} \sum_{k \in \mathcal{A}_{s,w}} \tilde{P}^{(o)}_{k,s,w}. \tag{2}$$

For the performance component we use a log transform (to accommodate very small shares):

$$\tilde{P}^{(\text{perf})}_{i,s,w} = \log\Big(P^{(\text{perf})}_{i,s,w} + \varepsilon\Big), \qquad y^{(\text{perf})}_{i,s,w} = \tilde{P}^{(\text{perf})}_{i,s,w} - \frac{1}{|\mathcal{A}_{s,w}|} \sum_{k \in \mathcal{A}_{s,w}} \tilde{P}^{(\text{perf})}_{k,s,w}. \tag{3}$$

1.2 Explanatory factors (all available celebrity features)

We include all celebrity features present in the panel: age during season (`celebrity_age_during_season`), industry (`celebrity_industry`), home state (`celebrity_homestate`), and home-country region (`celebrity_homecountry_region`). Age is standardized:

$$\text{age\_z}_{i,s} = \frac{\text{age}_{i,s} - \overline{\text{age}}}{\text{sd}(\text{age})}. \tag{4}$$

Industries and states are pooled by minimum frequency, and regions are top-N pooled, yielding grouped categories ind_i, state_i, reg_i. The professional dancer identity is pro_i (`ballroom_partner`).

1.3 Model specification

For each outcome $o \in \{J, \text{tot}, \star, \text{pro}, \text{perf}\}$ we fit:

$$y^{(o)}_{i,s,w} = \beta^{(o)}_{\text{age}} \, \text{age\_z}_{i,s} + \alpha^{(o)}_{\text{ind}(i)} + \alpha^{(o)}_{\text{state}(i)} + \alpha^{(o)}_{\text{reg}(i)} + \gamma^{(o)}_{\text{pro}(i)} + \varepsilon^{(o)}_{i,s,w}, \tag{5}$$

with category fixed effects α and professional fixed effects γ. Standard errors are clustered at the season–celebrity level (same celebrity across weeks).

1.4 Evaluation metrics

(i) Incremental explanatory power of professionals. Define $R^2_{\text{no\_pro}}(o)$ from the model without $\gamma^{(o)}_{\text{pro}(i)}$, and $R^2_{\text{full}}(o)$ from the full model. Then

$$\Delta R^2_{\text{pro}}(o) = R^2_{\text{full}}(o) - R^2_{\text{no\_pro}}(o). \tag{6}$$

(ii) Factor importance via drop-one ΔR^2. For factor $f \in \{\text{age}, \text{industry}, \text{state}, \text{region}, \text{pro}\}$, let $R^2_{-f}(o)$ be the R^2 from the model dropping only factor f, fit on the same complete-case sample as the full model. Define

$$\Delta R^2_{\text{drop}}(o, f) = R^2_{\text{full}}(o) - R^2_{-f}(o). \tag{7}$$

(iii) Cross-outcome alignment (same-way test). For a categorical factor f with category effects $\theta^{(o)}_{f,c}$ (centered by category frequency), define

$$\text{Align}_f(o) = \text{Corr}\left(\theta^{(J)}_{f,\cdot},\ \theta^{(o)}_{f,\cdot}\right). \tag{8}$$

(iv) Age effect as a per-10-year multiplier. Let σ_{age} be the age standard deviation (years). The multiplicative change per +10 years is

$$\text{Mult}^{(o)}_{\text{age}}(+10) = \exp\left(\beta^{(o)}_{\text{age}} \cdot \frac{10}{\sigma_{\text{age}}}\right). \tag{9}$$

1.5 Results

1.5.1 How much do professionals add beyond celebrity features?

Across all outcomes, professionals contribute materially beyond celebrity features:

$$\Delta R^2_{\text{pro}}(J) = 0.0994, \quad \Delta R^2_{\text{pro}}(\text{tot}) = 0.0922, \quad \Delta R^2_{\text{pro}}(\star) = 0.0923, \tag{10}$$

$$\Delta R^2_{\text{pro}}(\text{pro}) = 0.0591, \quad \Delta R^2_{\text{pro}}(\text{perf}) = 0.0520, \tag{11}$$

with $R^2_{\text{full}}(J) = 0.3577$ and $R^2_{\text{full}}(\text{tot}) = 0.2487$. Therefore, professional identity explains roughly 5%–10% absolute R^2 beyond all celebrity features.

1.5.2 Do factors affect judges and fans in the same way?

Professional effects align moderately between judges and fans total:

$$\text{Align}_{\text{pro}}(\text{tot}) = 0.7546, \tag{12}$$

and align most strongly with the performance component:

$$\text{Align}_{\text{pro}}(\text{perf}) = 0.8933, \tag{13}$$

while alignment is weakest on the pro-fixed component:

$$\text{Align}_{\text{pro}}(\text{pro}) = 0.5292. \tag{14}$$

Industry effects show a similar pattern, with especially high alignment on the performance component:

$$\text{Align}_{\text{ind}}(\text{perf}) = 0.9650. \tag{15}$$

State effects align less tightly overall, particularly on the pro-fixed component:

$$\text{Align}_{\text{state}}(\text{pro}) = 0.4907. \tag{16}$$

Region contributes negligible explanatory power (drop-one $\Delta R^2 \approx 0$) and is not substantively interpreted.

1.5.3 Factor importance (all celebrity features + professionals)

Drop-one ΔR^2 shows the relative contribution of each factor.

Judges. The two dominant factors are professionals and age:

$$\Delta R^2_{\text{drop}}(J, \text{pro}) = 0.0994, \qquad \Delta R^2_{\text{drop}}(J, \text{age}) = 0.0885, \tag{17}$$

followed by industry and state:

$$\Delta R^2_{\text{drop}}(J, \text{industry}) = 0.0320, \qquad \Delta R^2_{\text{drop}}(J, \text{state}) = 0.0281, \tag{18}$$

and region is effectively zero.

Fans (total and components). For FansTotal and FansStar, professionals dominate, with age and state next:

$$\Delta R^2_{\text{drop}}(\text{tot}, \text{pro}) = 0.0922, \quad \Delta R^2_{\text{drop}}(\text{tot}, \text{age}) = 0.0393, \quad \Delta R^2_{\text{drop}}(\text{tot}, \text{state}) = 0.0330, \tag{19}$$

$$\Delta R^2_{\text{drop}}(\star, \text{pro}) = 0.0923, \quad \Delta R^2_{\text{drop}}(\star, \text{age}) = 0.0412, \quad \Delta R^2_{\text{drop}}(\star, \text{state}) = 0.0333. \tag{20}$$

FansPro is driven mainly by professional identity:

$$\Delta R^2_{\text{drop}}(\text{pro}, \text{pro dancer}) = 0.0591, \tag{21}$$

while FansPerf is jointly driven by age and professionals:

$$\Delta R^2_{\text{drop}}(\text{perf}, \text{age}) = 0.0585, \qquad \Delta R^2_{\text{drop}}(\text{perf}, \text{pro}) = 0.0520. \tag{22}$$

1.5.4 Age effect across outcomes

Age penalizes fans far more strongly than judges. With $\sigma_{\text{age}} = 12.8691$ years, the per-10-year multipliers are:

$$\text{Mult}^{(J)}_{\text{age}}(+10) = 0.9538, \quad \text{Mult}^{(\text{tot})}_{\text{age}}(+10) = 0.6681, \quad \text{Mult}^{(\star)}_{\text{age}}(+10) = 0.6595, \tag{23}$$

$$\text{Mult}^{(\text{pro})}_{\text{age}}(+10) = 0.8464, \quad \text{Mult}^{(\text{perf})}_{\text{age}}(+10) = 0.6367. \tag{24}$$

Thus, holding all other features fixed, a 10-year increase corresponds to $\approx 33\%$ lower fan total-share odds but only $\approx 4.6\%$ lower judge-share odds.

1.5.5 Mechanism: how professionals shape fan totals

Let $\gamma_{\text{pro}}^{(\text{tot})}$ be the estimated professional effect on FansTotal and similarly $\gamma_{\text{pro}}^{(\star)}, \gamma_{\text{pro}}^{(\text{pro})}, \gamma_{\text{pro}}^{(\text{perf})}$ for components. Among stable professionals (at least 30 observed weeks), the correlations are:

$$\operatorname{Corr}\left(\gamma_{\text{pro}}^{(\text{tot})}, \gamma_{\text{pro}}^{(\star)}\right) = 0.9997, \quad \operatorname{Corr}\left(\gamma_{\text{pro}}^{(\text{tot})}, \gamma_{\text{pro}}^{(\text{pro})}\right) = 0.8953, \quad \operatorname{Corr}\left(\gamma_{\text{pro}}^{(\text{tot})}, \gamma_{\text{pro}}^{(\text{perf})}\right) = 0.7266. \tag{25}$$

Empirically, the decomposition is dominated by the celebrity fan-base component:

$$\mathbb{E}\left[\frac{P^{(\star)}}{P^{(\text{tot})}}\right] \approx 0.947, \qquad \mathbb{E}\left[\frac{P^{(\text{pro})}}{P^{(\text{tot})}}\right] \approx 0.053, \tag{26}$$

so professional effects on FansTotal transmit primarily through the star-driven component.

1.6 Answer to the question

Professionals and celebrity features both matter, but with distinct signatures across judges and fans. Professionals add ≈ 0.05–0.10 absolute R^2 beyond all celebrity features across outcomes, and are the single largest contributor for FansTotal/FansStar and FansPro. Age is the strongest celebrity feature overall and penalizes fan outcomes substantially more than judges (per-10-year multipliers: 0.668 for FansTotal vs 0.954 for Judges). Industry and state effects are non-negligible, with moderate judges–fans alignment, while region contributes essentially zero incremental explanatory power in this dataset.